JPEG Image Compression

Rahul Garg, Varun Gulshan

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Basic Data Redundancies

- Interpixel Redundancy
- Psychovisual Redundancy
- Coding Redundancy

A Bit of Information Theory

Entropy of a source : Average amount of information obtained by observing a single source output.

$$H(z) = -\sum_{i=1}^{N} P(a_i) log P(a_i)$$

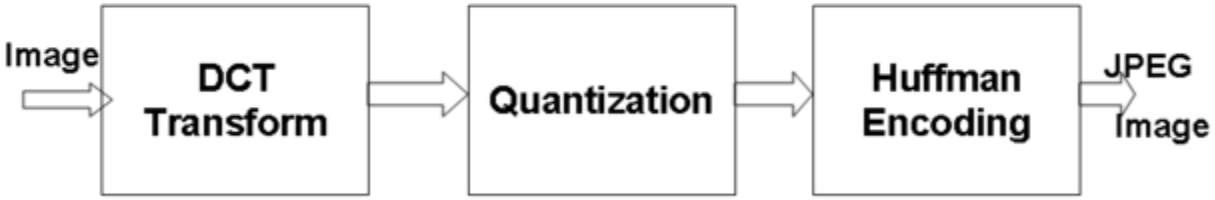
where $(a_1, a_2, ..., a_N)$ forms the set of possible outputs.*Shannon's First Theorem* states that

$$min(L_{avg}) = H(z)$$

where L_{avg} is average code word length per source symbol.

Encoding Process

General Steps in Image Compression



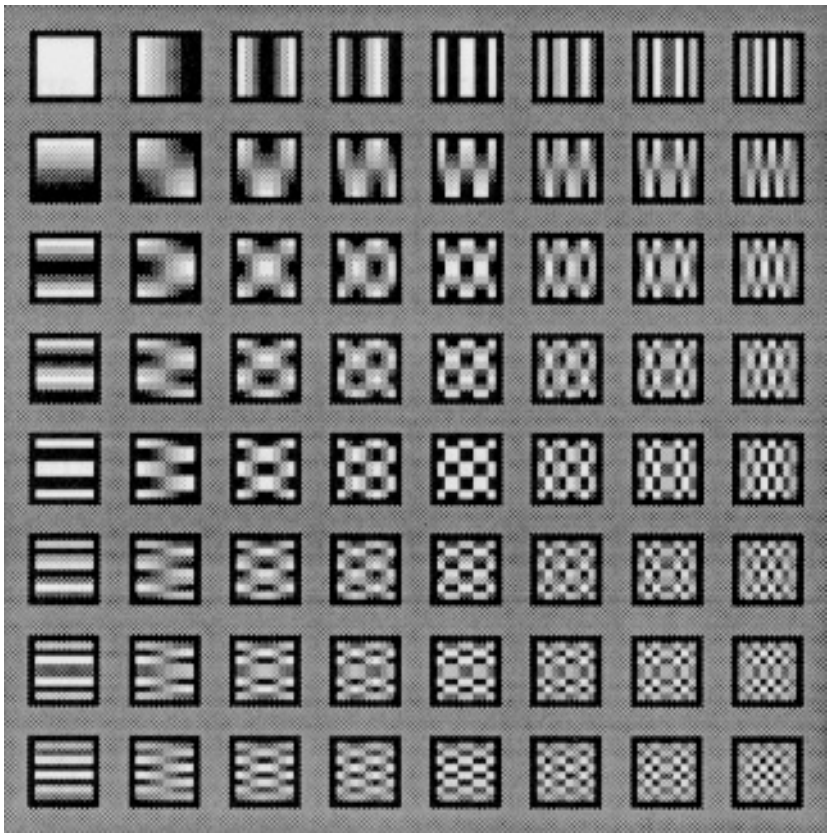
Steps in JPEG Image Compression

Handling Redundancies

- Interpixel Redundancy: Captured by the DCT coefficients(more zeros in coeffs).
- Psychovisual Redundancy: Captured by the high frequency DCT coefficients.
- Coding Redundancy: Captured by variable length codes.

The DCT transform

- Break up image into 8x8 image blocks.
- Change the basis for representing the block image.



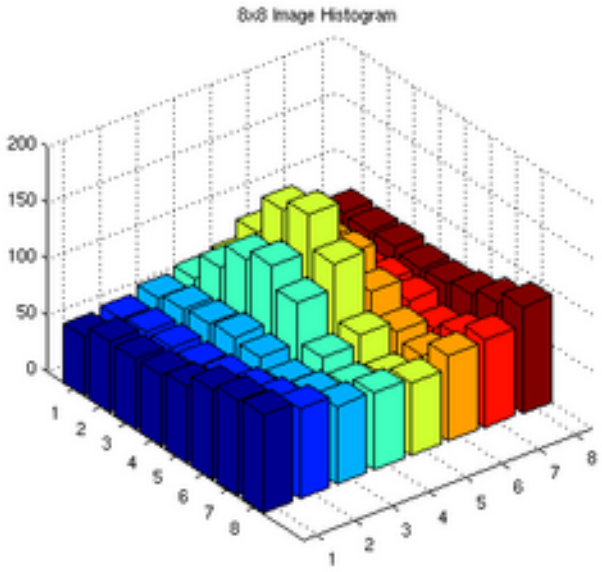
The $(u,v)^{th}$ DCT coefficient is given by the dot product

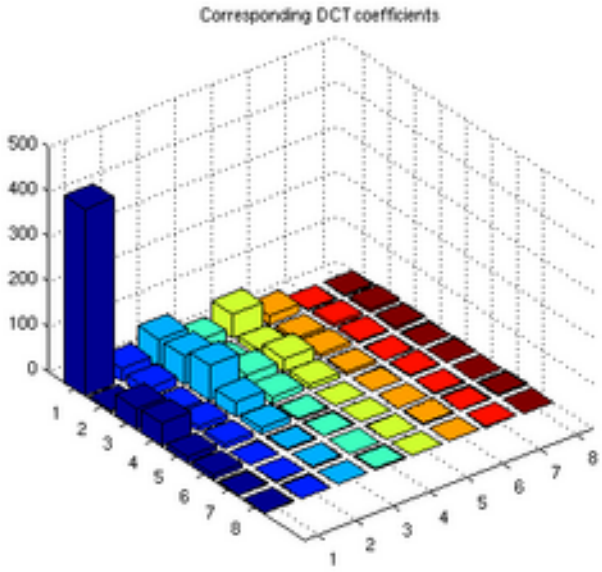
$$dct\,(u,v) = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} image\,(x,y) * h\,(x,y,u,v)$$

where

$$h\,(x,y,u,v) = \alpha(u)\alpha(v)cos\left[\frac{(2x+1)u\pi}{2N}\right]cos\left[\frac{(2y+1)v\pi}{2N}\right]$$

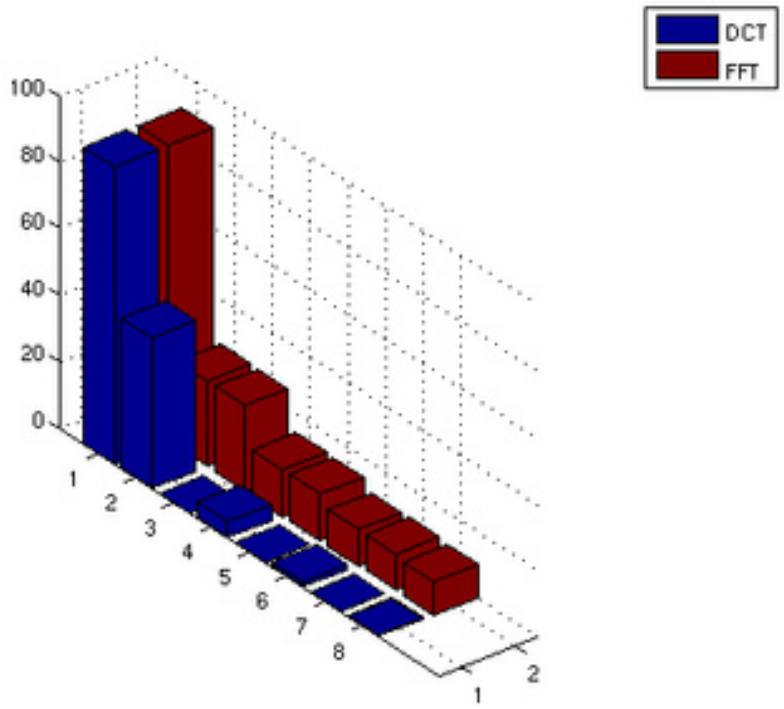
Information packing in DCT coeffs



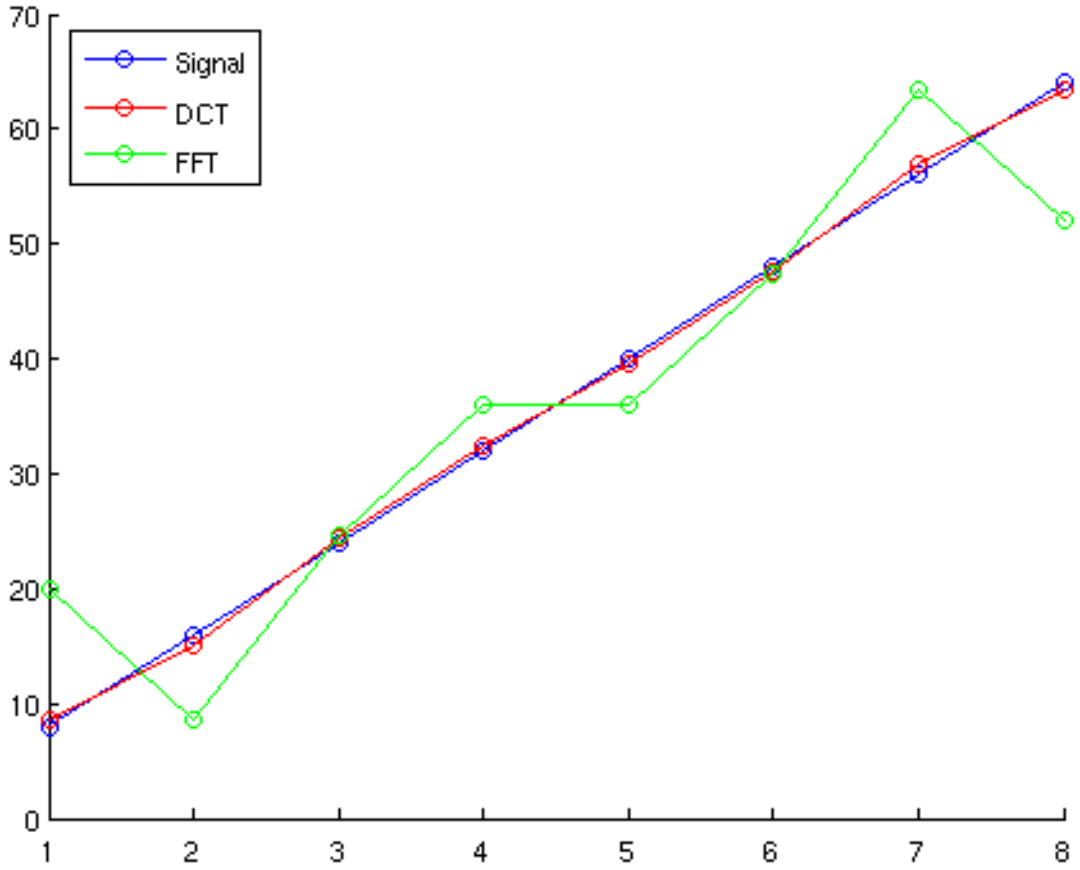


DCT vs FT

Comparison of packing for a one dimensional test signal:$[8, 16, 32, 40, 48, 56, 64]$



Effect of truncation



Result on images

FT

DCT

Quantization

- Equivalent to truncating coefficients corresponding to high frequency.
- Lossy step in JPEG compression.
- Quantization values specified in a 8x8 Q-table.
- Table based on heuristically determined perceptual importance of each coefficient.
- Image quality can be controlled by scaling Q table.

Original Image

52	55	61	66	70	61	64	73
63	59	66	90	109	85	69	72
62	59	68	113	144	104	66	73
63	58	71	122	154	106	70	69
67	61	68	104	126	88	68	70
79	65	60	70	77	68	58	75
85	71	64	59	55	61	65	83
87	79	69	68	65	76	78	94

DCT Transform

-414	-29	-62	25	55	-20	-1	2
6	-21	-62	8	12	-7	-6	7
-46	8	77	-26	-30	10	6	-5
-49	12	34	-14	-10	6	1	1
11	-8	-12	-2	0	1	-5	2
-10	1	3	-3	0	0	2	0
-3	-1	1	0	1	-4	2	-3
-1	-1	0	-3	0	0	-1	0

Quantization

-26	-3	-6	2	2	-1	0	0
1	-2	-4	0	0	0	0	0
-3	1	5	-1	-1	0	0	0
-4	1	2	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Standard JPEG Q-table

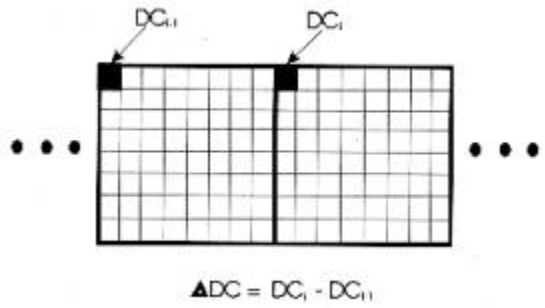
Coding of the DCT coefficients

Done in two parts

- The DC coefficient $(u, v) = (0, 0)$ is coded using difference encoding.
- The AC coefficients(the rest of the coefficients) are run length encoded and then huffman coded.

Coding the DC coefficients

Step 1: Take difference with the DC coefficient of the previous block.



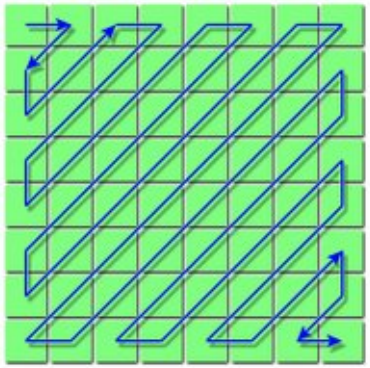
Step 2: Differences are encoded using variable length codes.These variable length codes are standardized for jpeg.

DC coeffs	-869	→	-861	→	-876	→	-933	→	-867	→	-845	→	-863	...
Differences		8		-15		-57		66		22		-18		...

Codes for differences are looked up from a table.

Coding the AC coefficients

Step 1: Convert the AC coefficients into a sequence by traversing the coefficients in a zig-zag way.



Zig-zag parse clusters the zeros together.

Example zig zag parse

-26	-3	-6	2	2	-1	0	0
1	-2	-4	0	0	0	0	0
-3	1	5	-1	-1	0	0	0
-4	1	2	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

-26 -3 1 -3 -2 -6 2 -4 1 -4 1 1

5 0 2 0 0 -1 2 0 0 0 0 0 -1 -1 EOB

Step 2: Non zero AC coefficients are divided into categories.

Category	AC Value
1	-1, 1
2	-3, -2, 2, 3
3	-7..-4, 4..7
4	-15..-8, 8..15
.	.
.	.
.	.
10	-1023..-512, 512..1023

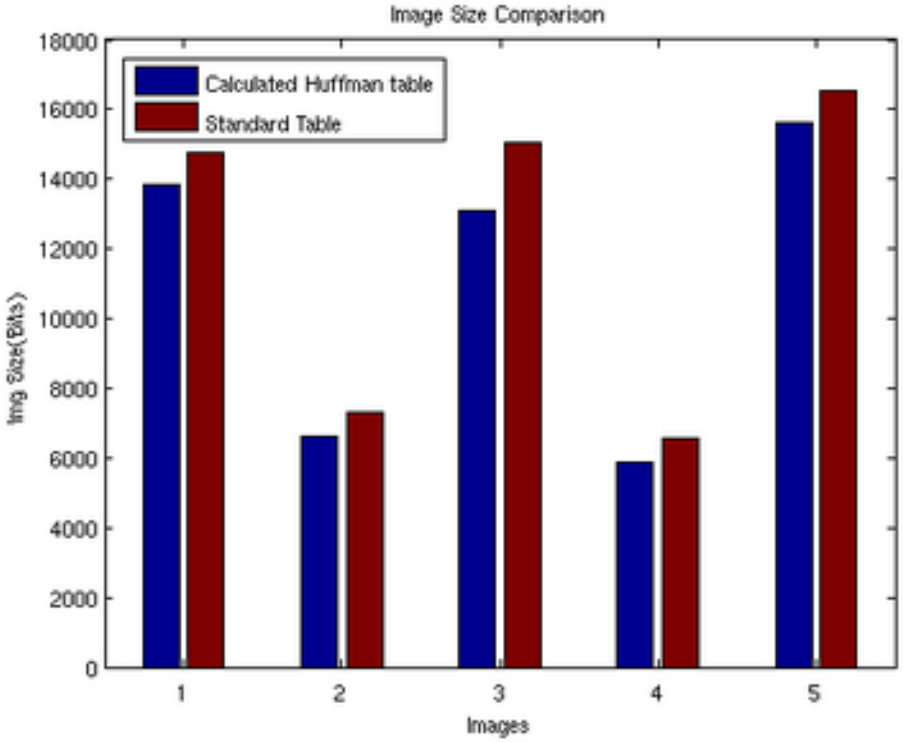
Figure 1: AC categories

The sequence is then run length coded.

Original AC coeff	-26	-3	1	-3	-2	-6	2	-4	1	-4	1	1	5	0
	2	0	0	-1	2	0	0	0	0	0	-1	-1	EOB	
						⇓								
Category codes	5	2	1	2	2	3	2	3	1	3	1	1	3	-
	2	-	-	1	2	-	-	-	-	-	1	1	EOB	
						⇓								
Run length codes	0/5	0/2	0/1	0/2	0/2	0/3	0/2	0/3	0/1	0/3	0/1	0/1	0/3	
	1/2			2/1	0/2						5/1	0/1	EOB	

Run length codes form our symbol set for coding. They are coded using a standardized table. Hence, code for each non zero AC coefficient is composed of a basecode (corresponding to runlength/category) and a code corresponding to offset in the category.

Standard tables vs Optimized tables



JPEG Variants

- Progressive JPEG : Image stored in series of scans with image becoming sharper with each scan.
- Lossless JPEG : Utilizes Predictive Differential Coding method.
- JPEG 2000 : Latest version of JPEG relying on wavelet transformation instead of DCT.

References

- Digital Image Processing by R.C. Gonzalez and R.E. Woods
- http://www.jpeg.org
- http://cnx.rice.edu
- http://www.datacompression.info
- http://www.cs.sfu.ca
- http://www.wikipedia.org